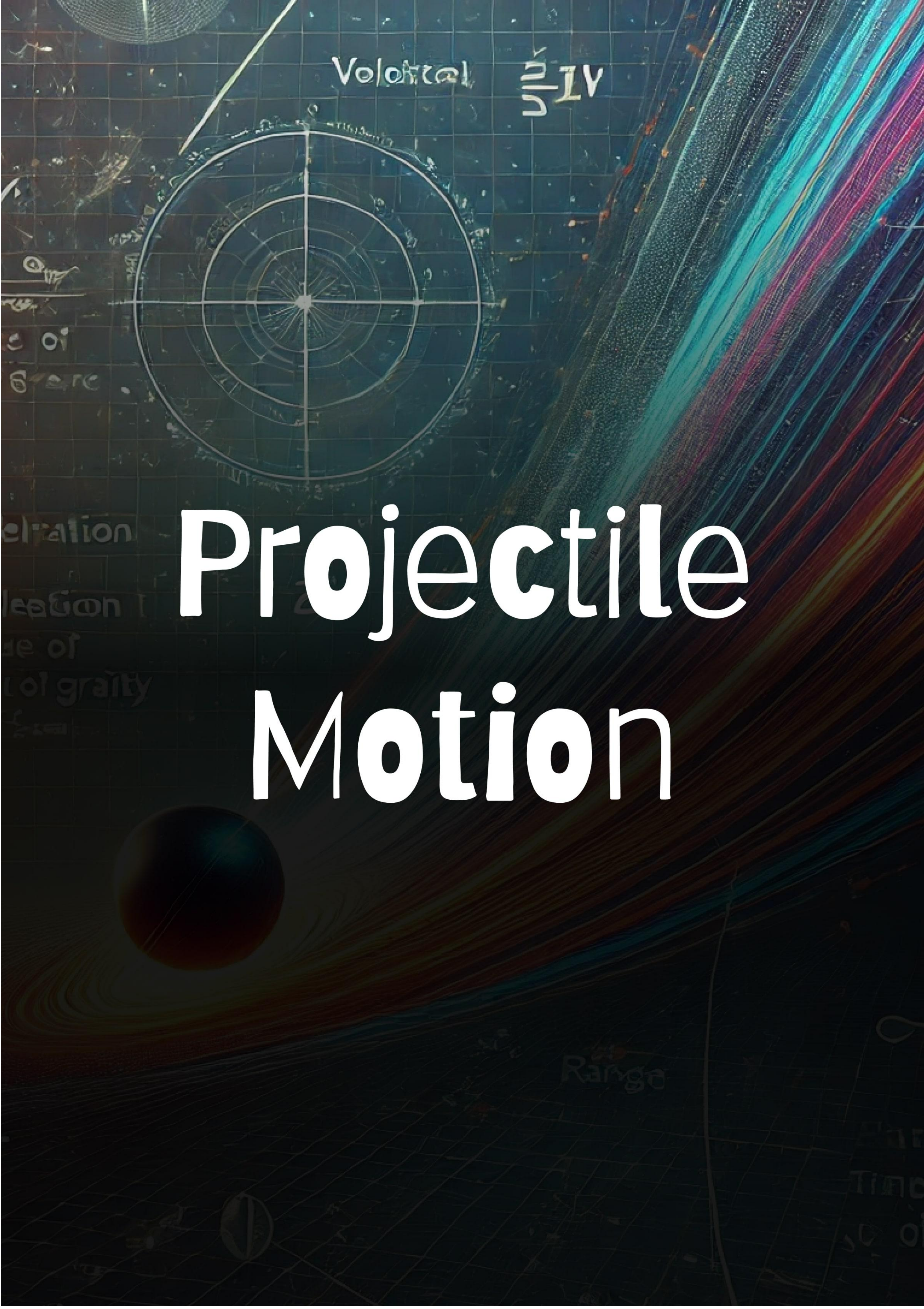


Projectile Motion

Projectile motion

Any stone launched under the effect of gravitation field neglecting the air drag effect, forms the standard projectile motion.

Q.) A stone is projected with a speed u at an angle θ with x-axis in the x-y plane from the origin.

(i) Find its time of flight (assuming x-axis to be the ground level)

(ii) Find the range of projectile.

(iii) Find the max. height of projectile.

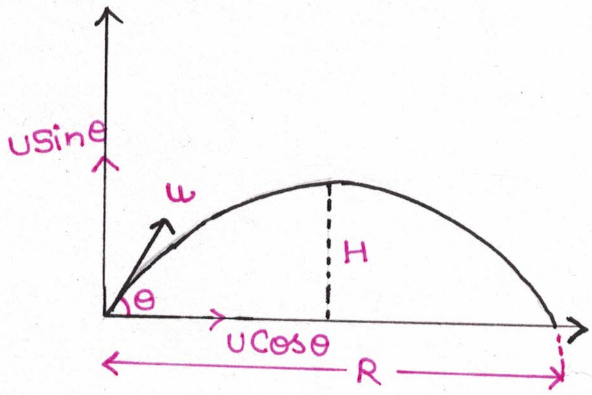
(iv) For what projection angle is the range max.

(v) Show that the range is same for complimentary angles of projection.

(vi) What is the trajectory equation.

$u_x = u\cos\theta \quad , a_x = 0$

$u_y = u\sin\theta \quad , a_y = -g$



(i) Let time of flight be T

@ T, $S_y = 0$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$u\sin\theta\, T - \frac{1}{2} g T^2 = 0$$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

(ii) $S_x = u_x t + \frac{1}{2} a_x t^2$

@ $t = T$, $S_x = R$

$$R = u\cos\theta \cdot T + 0$$

$$R = \frac{u\cos\theta}{g} \times 2u\sin\theta$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

(iii) $v_y^2 - u_y^2 = 2a_y \cdot S_y$

@ $S_y = +H$, $v_y = 0$

$0 \cancel{-} u^2\sin^2\theta = \cancel{-}2gH$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

(iv) For max. range

$\sin 2\theta = 1$

$2\theta = 90^\circ$

$$\boxed{\theta = 45^\circ}$$

(v) Let $\phi = 90^\circ - \theta$

$$R_\phi = \frac{u^2 \sin 2(90^\circ - \theta)}{g}$$

$$R_\phi = \frac{u^2 \sin(180^\circ - 2\theta)}{g}$$

$$R_\phi = \frac{u^2 \sin 2\theta}{g} = R_\theta$$

$$\therefore \quad \boxed{R_{90^\circ-\theta} = R_\theta}$$

(vi) $x = u_x t + \frac{1}{2}a_x t^2$ —①

$x = u\cos\theta t$

$y = u_y t + \frac{1}{2}a_y t^2 + 0$

$y = u\sin\theta \cdot t - \frac{1}{2}gt^2$ —②

$$t = \frac{x}{u\cos\theta}$$

using in eq.②

$$y = \cancel{u}\sin\theta \times \frac{x}{\cancel{u}\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta\left[1 - \frac{1}{2}\frac{gx^2}{u^2\cos^{\cancel{2}}\theta \times \frac{\sin\theta}{\cancel{\cos\theta}}}\right]$$

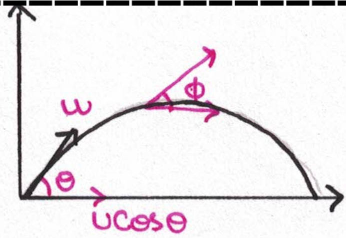
$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

Q.) In the previous question what will be the speed of the stone when it is flying at a general angle (say ϕ) with the horizontal

$v\cos\phi = u\cos\theta$

$v\cos\phi = u\cos\theta$

$$\boxed{v = \frac{u\cos\theta}{\cos\phi}}$$



Q. A ball is projected from the floor at an angle of 60° at 15m/s. Find (i) Time of flight, (ii) Range, (iii) Max. height, (iv) Its height when it is 5m away from the point of projection. (v) Will it be able to hit a vertical wall, 5m away (Explain).

(iv) $5 = \frac{15}{2} \times t$

$t = \frac{10}{15}$ s

$H = \frac{\cancel{15}}{\cancel{2}} \times \sqrt{3} \times \frac{\cancel{10}^{5}}{\cancel{15}} - \frac{1}{\cancel{2}} \times \cancel{10}^{5} \times \frac{100}{225}$

$= 5\sqrt{3} - \frac{500}{225}$

$= 5\sqrt{3} - \frac{20}{9} \simeq 6.44m$

(i) $T = \frac{\cancel{2} \times \cancel{15}^{3} \times \sqrt{3}/2}{\cancel{10}_{\cancel{5}}}$

$T = \frac{3\sqrt{3}}{2}$ sec.

(ii) $R = \frac{15 \times 15 \times \sqrt{3}}{10 \times 2} = \frac{225\sqrt{3}}{20}$

$R = \frac{45\sqrt{3}}{4}$ m

(iii) $H = \frac{15 \times 15 \times 3}{20 \times 4} = \frac{135}{16}$

(v) It will hit wall if its height is 6.44m.

Q. In previous question, find velocity of ball when it is an angle 30° with horizontal.

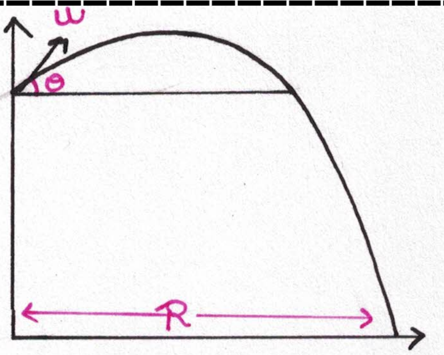
$v = \frac{u\cos 60^\circ}{\cos 30^\circ} = \frac{15}{\sqrt{3}}$

$v = 5\sqrt{3}$ m/sec.

PROJECTION FROM A TOWER

Equation of Trajectory

$$\boxed{y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u\cos^2\theta}}$$



Q.) A projectile is launched with a speed u at an angle θ with the horizontal as shown in fig. Find
(i) Time of flight
(ii) The range of projectile

$u_x = u\cos\theta$

$u_y = u\sin\theta$

$S_y = -H \quad @ \quad t = T$

$a_y = -g$

$$-H = u\sin\theta . T - \frac{1}{2}gt^2$$

$$gT^2 - 2u\sin\theta . T - 2H = 0$$

$$T = \frac{2u\sin\theta \pm \sqrt{4u^2\sin^2\theta + 8Hg}}{2g}$$

$$T = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta + 2gH}}{g}$$

$$R = u_x T = u\cos\theta . T$$

Q.) A stone is projected horizontally from a tower of height 20m at a speed of 10m/s. Find (i) The range, (ii) Time of flight, (iii) angle with vertical at which the stone hits the ground.

(i) $\cancel{-}40 = \cancel{-}\frac{1}{2}gT^2$

$$\frac{2\cancel{0}}{1\cancel{0}} = T^2$$

$$T = 2 \text{ sec}$$

(ii) $R = 10 \times 2 = 20 \text{ m.}$

(iii) $\tan\alpha = \dfrac{V_x}{V_y}$

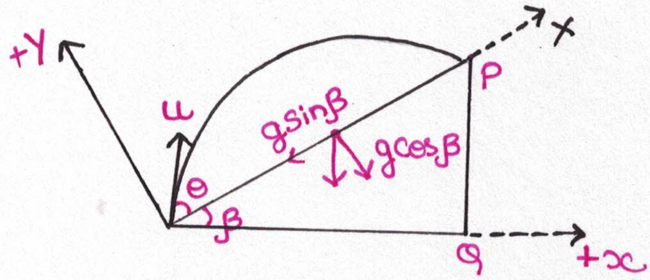
$$V_y = -10 \times 2 = -20$$

$$\alpha = \tan^{-1}\left(-\frac{1}{2}\right)$$

NOTE: To find speed at the ground, the angle of projection is not required.

PROJECTION ON AN INCLINED PLANE

Q.1 A stone is launched up and inclined plane of inclination β at an angle θ with the horizontal as shown in the fig. Find (i) Time of flight, (ii) Range of stone, (iii) The value of θ for which the range is maximum.



$u_x = u\cos(\theta-\beta)$

$u_y = u\sin(\theta-\beta)$

$a_x = -g\sin\beta$

$a_y = -g\cos\beta$

@ $t = T$, $S_y = 0$

$$0 = u\sin(\theta-\beta)T - \frac{1}{2}g\cos(\beta)T^2$$

$$\boxed{T = \frac{2u\sin(\theta-\beta)}{g\cos\beta}}$$

(ii) $OQ = u_x T$

$R = OP = OQ \cdot \sec\beta$

$$R = u\cos\theta \times \frac{2u\sin(\theta-\beta)}{g\cos\beta} \times \sec\beta$$

$$\boxed{R = \frac{2u^2\sin(\theta-\beta)\cos\theta}{g\cos^2\beta}}$$

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\theta-\beta) - \sin\beta\right] \quad (2\sin A\cdot\cos B)$$

For max. range

$\sin(2\theta-\beta) = 1$

$2\theta - \beta = 90^\circ$

$$\boxed{\theta = 45^\circ + \frac{\beta}{2}}$$

Q. In the previous que., what will be the max. distance of the projectile from the inclined plane.

$$V_y = 0 \text{ when } S_y = H$$

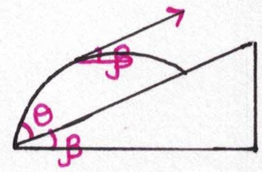
$$0 - u_y^2 = 2gH$$

$$H = \frac{u^2 \sin^2(\theta - \beta)}{2g \cos\beta}$$

Q. In the previous problem, what will be the speed of the stone when it is flying parallel to the inclined plane.

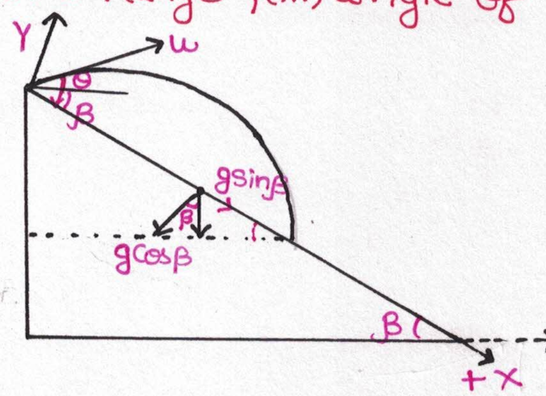
$$v \cos\beta = u \cos\theta$$

$$v = \frac{u \cos\theta}{\cos\beta}$$



PROJECTION DOWN ON AN INCLINED PLANE

Q. A stone is thrown down an inclined plane with angle θ with horizontal. Find (i) its time of flight, (ii) its Range, (iii) angle of projection. 'R' is maximum.



$$u_x = u\cos(\theta + \beta)$$

$$u_y = u\sin(\theta + \beta)$$

$$a_x = g\sin\beta$$

$$a_y = -g\cos\beta$$

$$R = u\cos\theta \cdot T \cdot \sec\beta$$

$$R = \frac{u\cos\theta \cdot 2u\sin(\theta+\beta)}{g\cos^2\beta}$$

$$T = \frac{2u\sin(\theta+\beta)}{g\cos\beta}$$

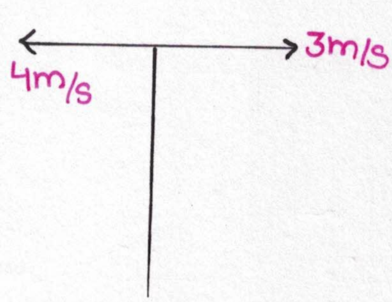
For R_{max}.

$$\theta = \left(45^\circ - \frac{\beta}{2}\right)$$

Q. Two stones are thrown from a high tower 4 m/s and 3 m/s simultaneously.

(i) After how much time will their velocity vectors be mutually ⊥.

(ii) What will be the separation of stone at that time.



(i) $v_1 = 3\hat{i} - gt\hat{j}$

$$v_2 = -4\hat{i} - gt\hat{j}$$

$$v_1 \cdot v_2 = 0$$

$$-12 + g^2t^2 = 0$$

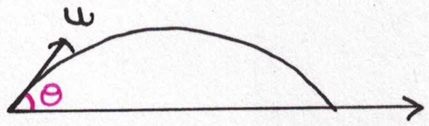
$$T = \frac{\sqrt{12}}{g}$$

(ii) Distance of separation $= 3T + 4T$

$$= \frac{3\sqrt{12}}{g} + \frac{4\sqrt{12}}{g}$$

$$= \frac{7\sqrt{12}}{g}$$

Q. A stone is launched with a speed U at θ. After how much time will it be moving ⊥ to the initial launch direction. What should be the minimum value of θ for which such an event may occur for ground to ground projectile.



(i) $\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v}\cdot\vec{u} = 0$

$$u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta \cdot gT = 0$$

$$u^2 - u\sin\theta\, gT = 0$$

$$T = \frac{u}{g\sin\theta}$$

(ii) $\dfrac{u}{g\sin\theta} \leqslant \dfrac{2u\sin\theta}{g}$

$$\sin^2\theta \geqslant \frac{1}{2}$$

$$\sin\theta \geqslant 1/\sqrt{2}$$

$$\theta \geqslant 45^\circ$$

Q. For what angle of θ range is same as height.

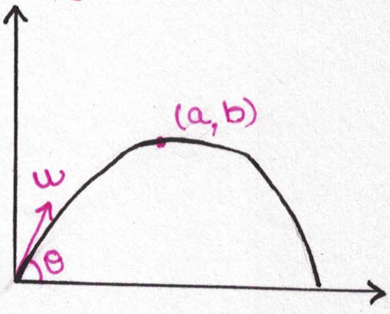
$$\frac{\cancel{u^2}\sin^2\theta}{2\cancel{g}} = \frac{\cancel{u^2}\sin 2\theta}{\cancel{g}}$$

$$\sin\theta = 2\cos\theta \times 2$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1}(4)$$

Q. A particle is to be launched from the origin so as to pass from (a, b). What will be the minimum velocity to accomplish thi



$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

$$b = a\tan\theta - \frac{1}{2}g\,\frac{a^2(1+\tan^2\theta)}{u^2}$$

$$2u^2 b = 2u^2 a\tan\theta - ga^2(1+\tan^2\theta)$$

$$2u^2b = 2u^2a\tan\theta - ga^2 + \tan^2\theta\, ga^2$$

$$ga^2\tan^2\theta - 2u^2a\tan\theta + 2u^2b + ga^2 = 0$$

$D \geq 0$ for real value of θ or $\tan\theta$

$$4u^4a^2 - 4(2u^2b + ga^2)\,ga^2 \geq 0$$

for min. value of u, it should be equal to 0. It is quadratic in u.

<u>Or</u>

let $q = u^2$ and $p = \tan\theta$

$$\therefore b = ap - \frac{ga^2(1+p^2)}{2q}$$

$$2qb = 2qap - ga^2(1+p^2)$$

For min. value of $dq/dp = 0$

$$\frac{d}{dp}(2qb) = \frac{d}{dp}\left(2qap - ga^2(1+p^2)\right)$$

$$0 = 2a[q] - ga^2(2p)$$

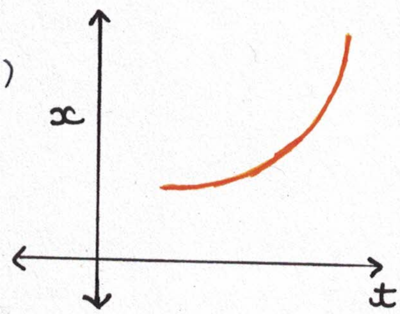
$$q = gap$$

$$\boxed{u^2 = ga\tan\theta}$$

GRAPHS FOR 1-DIMENSIONAL MOTIONS

Classification of Sections : Based on the first and second derivative, the sections of a graph can be divided into one of the seven categories :-

(i)

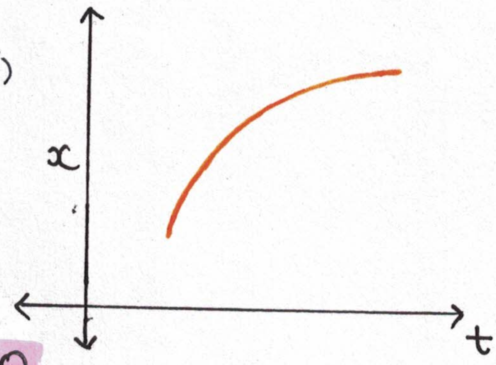


(Rising with upward curvature)

$\frac{dx}{dt} > 0$ (Slope)

$\frac{d^2x}{dt^2} > 0$ (Curvature)

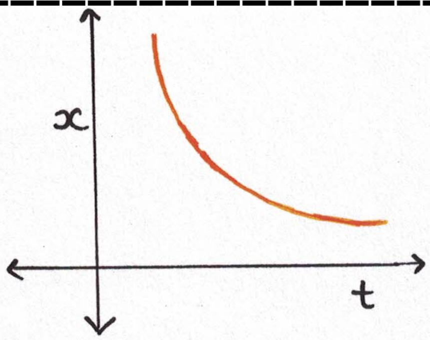
(ii)



(rising with downward curvature)

$\frac{dx}{dt} > 0$

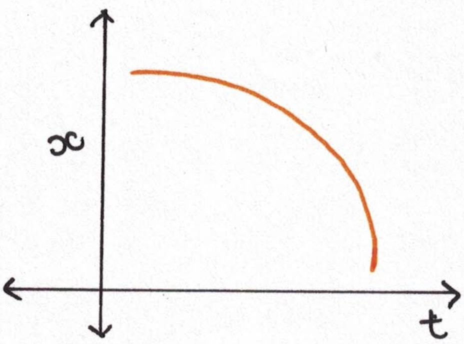
$\frac{d^2x}{dt^2} < 0$



(Falling with upward curvature)

$$\frac{dx}{dt} < 0$$

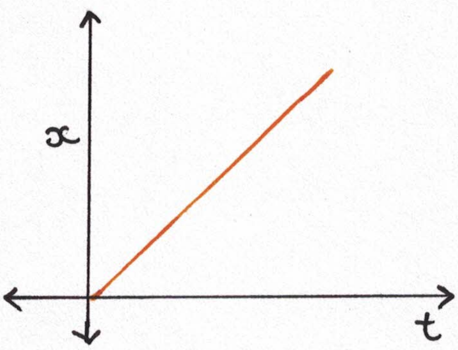
$$\frac{d^2x}{dt^2} > 0$$



(falling with downward curvature)

$$\frac{dx}{dt} < 0$$

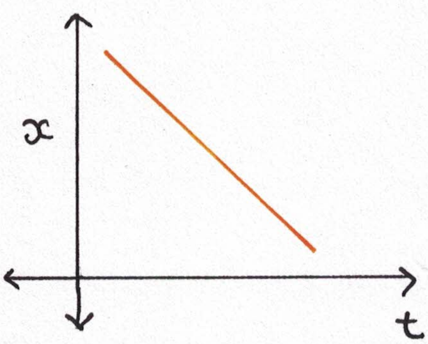
$$\frac{d^2x}{dt^2} < 0$$



(Rising with no curvature)

$$\frac{dx}{dt} > 0$$

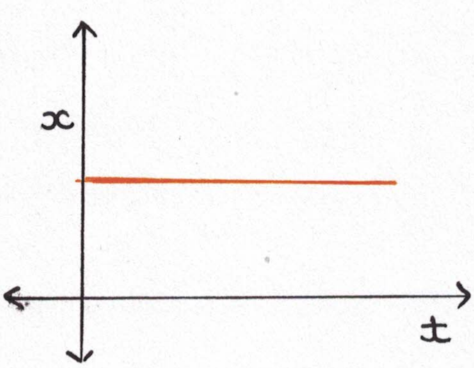
$$\frac{d^2x}{dt^2} = 0$$



(falling with no curvature)

$$\frac{dx}{dt} < 0$$

$$\frac{d^2x}{dt^2} = 0$$



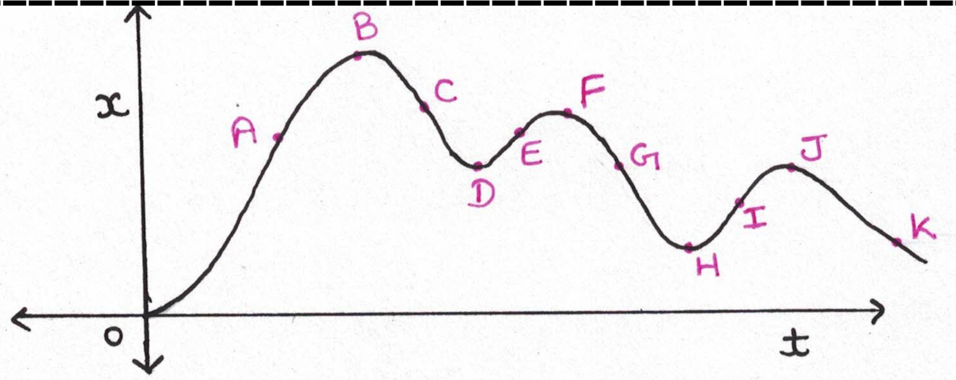
(Constant, no rise or fall)

$$\frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} = 0$$

51

Q.)



For the shown position versus time graph identify the sections where

(i) velocity is +ve
(ii) velocity is -ve
(iii) velocity is zero
(iv) acceleration is +ve
(v) acceleration is -ve

(i) O → B (excluding B)
D → F (excluding D & F)
H → J

(ii) B → D
F → H
J → K

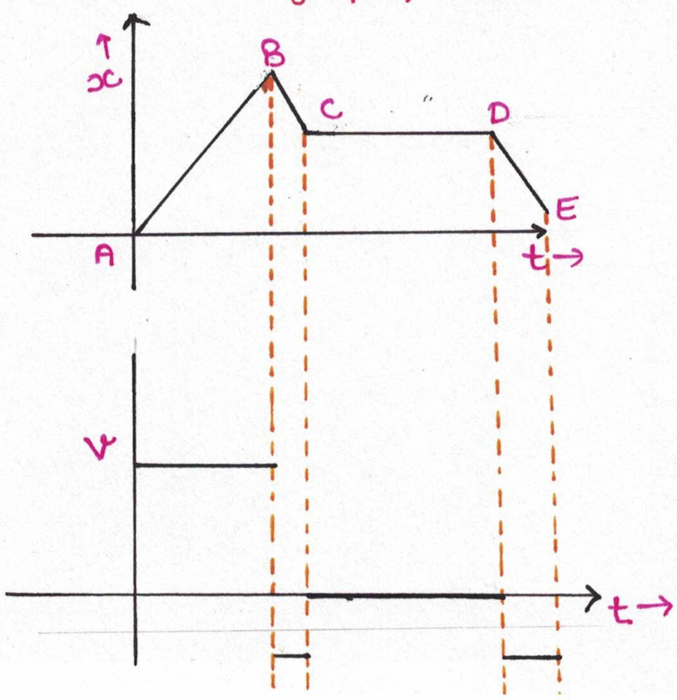
(iii) B, D, F, H, J

(iv) O → A , C → E, G → I

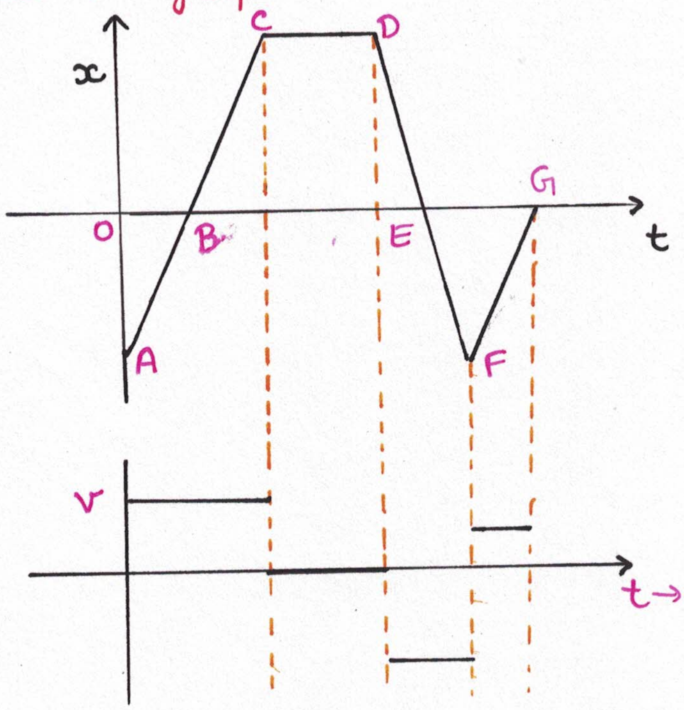
(v) A → C , E → G , I → K

DRAWING THE v-t GRAPH FROM THE s-t GRAPH

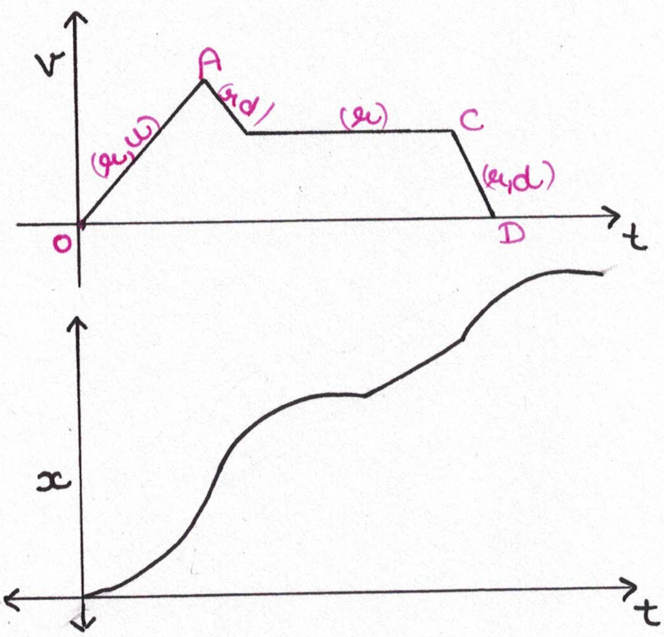
Q.) For the shown x-t graph, draw the corresponding v-t graph.



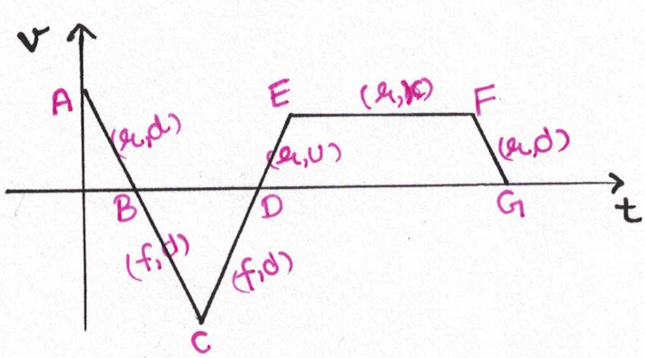
Q.) Draw v-t graph

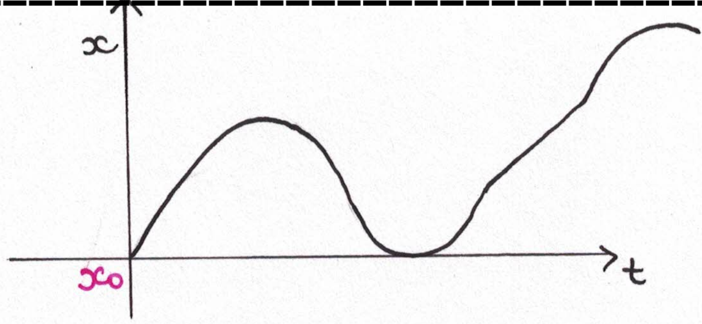


Q.) For the shown v-t graph draw the corresponding x-t graph if the initial position is x.



Q.) Draw x-t graph



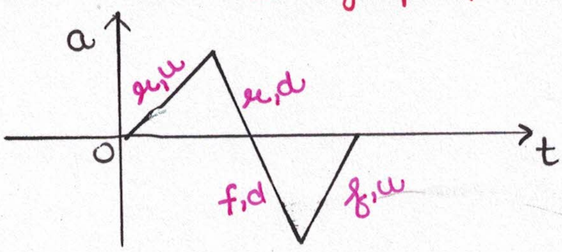


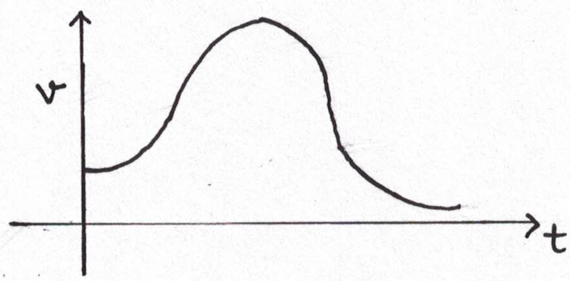
DRAWING THE v-t GRAPH FROM THE a-t GRAPH

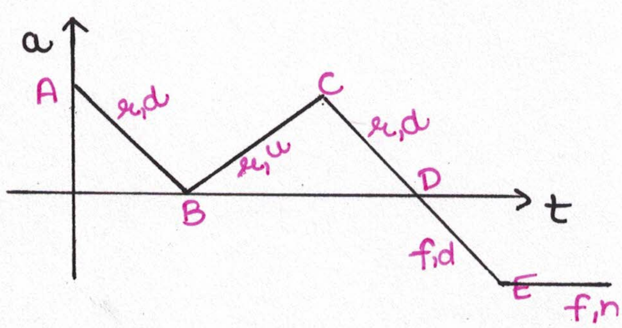
a-t to v-t is analogus to v-t to x-t graph due to the analogus differential equation i.e.

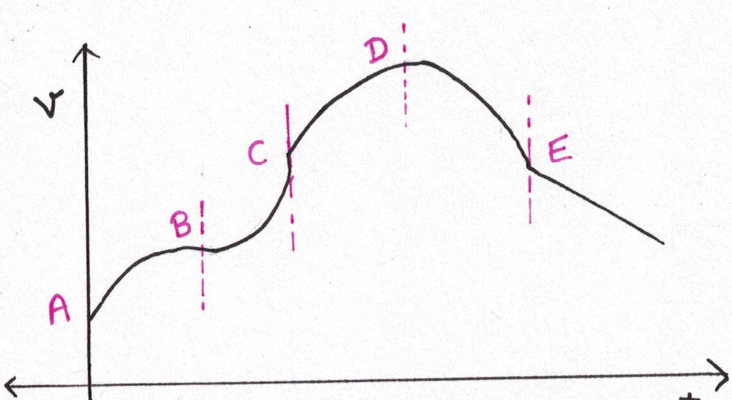
$$a = \frac{dv}{dt} \quad \text{and} \quad v = \frac{dx}{dt}$$

Q For the shown a-t graph, draw v-t graph

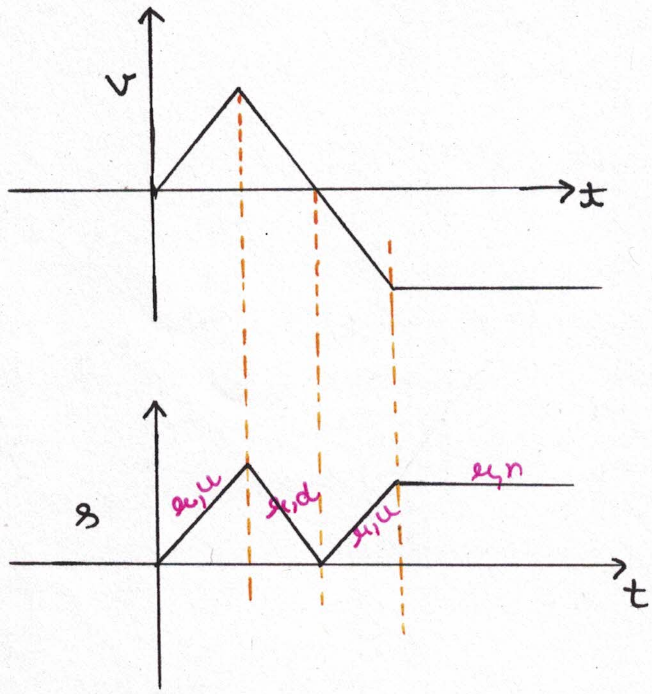






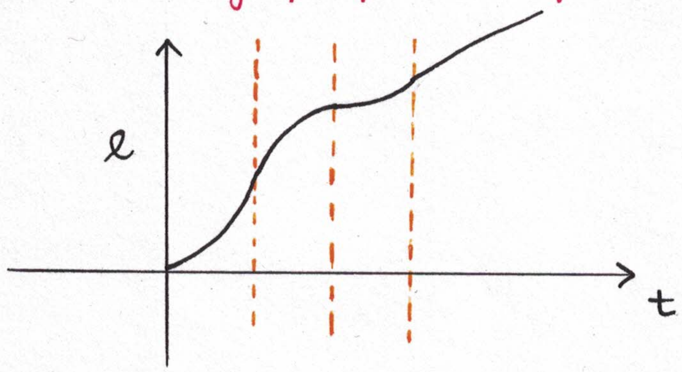


DRAWING SPEED TIME GRAPH OR |v|-t GRAPH



DRAWING l-t GRAPH FROM SPEED TIME GRAPH

Q.1) Draw lt graph from the previous |v|-t graph



PHYSICAL MEANING OF AREAS UNDER VARIOUS GRAPHS

1.) Area under the v-t graph represents the change in position or displacement.

$$v = dx/dt$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} v\,dt$$

$$S = x - x_0 = \int_{0}^{t} v \cdot dt$$

NOTE: Note that area above the x-axis is taken as positive and below the t-axis is taken as negative.

2.) Area under the a-t graph represents the change in velocity.

$$\frac{dv}{dt} = a$$

$$\int_u^v dv = \int_0^t a\,dt$$

$$v - u = \int_0^t a.dt$$

NOTE: The area below t-axis is taken as negative.

3.) Area under the |v|-t graph represents the distance travelled.

$$\frac{dl}{dt} = |v|$$

$$\int_0^l dl = \int_0^t |v|.dt$$

$$l = \int_0^t |v|.dt$$

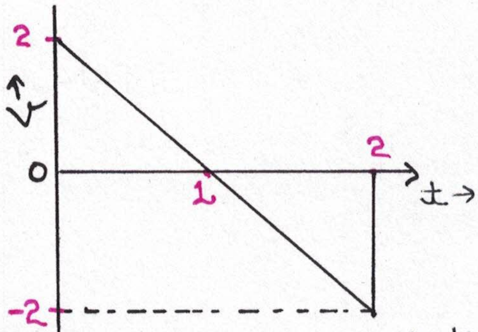
4.) Area under the a-s graph represents change in kinetic energy per unit mass.

$$a = v.\frac{dv}{ds}$$

$$\int_u^v v.dv = \int_0^s a.ds$$

(mass i 1Kg) $\leftarrow \frac{v^2}{2} - \frac{u^2}{2} = \int_0^s a.ds$

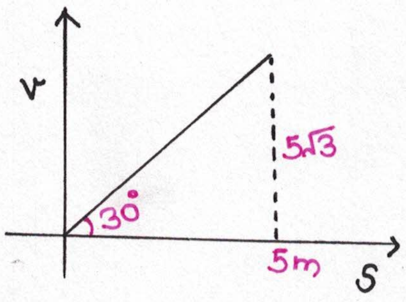
Q.2



Find
(i) Distance
(ii) Displacement
(iii) acceleration

(i) displacement = 0
(ii) distance = 2m
(iii) acceleration = $-2 m/s^2$

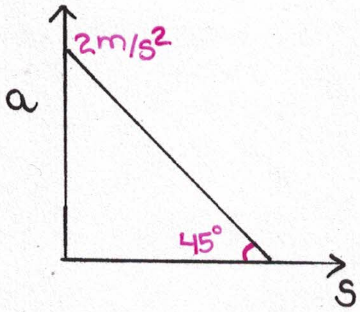
Q.)



Find acceleration

$$a = \frac{v\,dv}{ds}$$

$$a = \frac{5}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{5}{3}\, m/s^2$$

Q.) 

The initial speed of a particle is 2m/s. Find v

$$\frac{v^2}{2} - \frac{u^2}{2} = \frac{1}{\cancel{2}} \times \cancel{2} \times 2$$

$$v^2 - 4 = 4$$

$$v = \sqrt{8} = 2\sqrt{2} \text{ m/s}$$

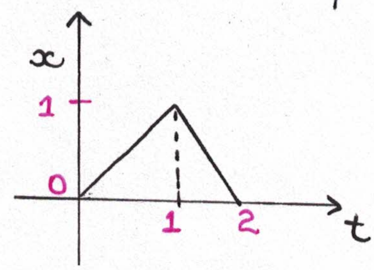
Q.) A particle travels along a straight line according to the equatⁿ $x = t(t-2)$. Find the distance travelled by the particle b/w time $t=0$ to $t = 2s$

$$\frac{dx}{dt} = 0 \qquad (v=0, \text{ when reversal takes place})$$

$$2t - 2 = 0$$

$$t = 1s$$

$$x = 2m$$



Q.) A particle travels acc. to $x = (t-1)(t-3)$. What is the distance travelled by a particle b/w $t=0$ & $t=4s$.

$t=0$, $x = 3m$
$t=1$, $x = 0m$
$t=2$, $x = 1m$
$t=3$, $x = 0m$
$t=4$, $x = 3m$

$x = 8m$

Or

$$\frac{dx}{dt} = 0$$

$$2t - 4 = 0$$

$$t = 4/2$$

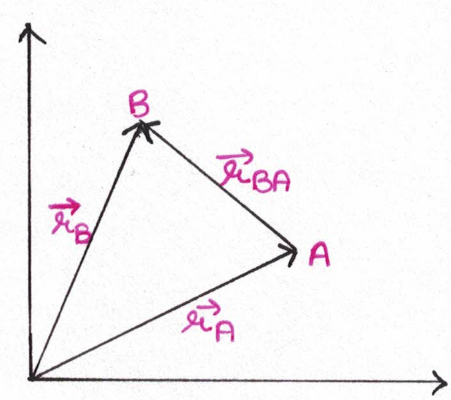
$$t = 2 \text{ sec.}$$

RELATIVE POSITION

Let A and B be two points having their position vectors $\vec{r}_A$ and $\vec{r}_B$ as shown in fig. Then the vector joining A to B ($\vec{r}_{BA}$) is called the position of B relative to A or position of B as seen from A.

$$\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$$

It can be easily seen from the fig. using triangle law.



RELATIVE VELOCITY

The ratio of change of relative position with respect to time is called relative velocity.

$$\frac{d}{dt}\vec{r}_{BA} = \frac{d}{dt}\vec{r}_B - \frac{d}{dt}\vec{r}_A$$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

RELATIVE ACCELERATION

The rate of change of relative velocity with respect to time is called relative acceleration.

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

CONDITION FOR COLLISSION OF UNIFORMLY ACCELERATED BODIES

$$r_A = \vec{r}_{0A} + \vec{u}_A + \frac{1}{2}\vec{a}_A t^2$$

$$r_B = \vec{r}_{0B} + \vec{v}_B t + \frac{1}{2}\vec{a}_B t^2$$

For Collision

$$\vec{r}_B = \vec{r}_A$$

$$\vec{r}_B - \vec{r}_A = 0$$

$$\vec{r}_{0BA} + \vec{u}_{BA} + \frac{1}{2}\vec{a}_{BA} t^2 = 0$$

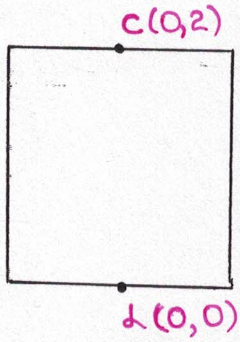
Q.) The floor to ceiling distance of an elevator is 2m. The elevator is moving upwards with an acceleration of $2 m/s^2$. when the speed of elevator is 5m/s a screw gets loose from the ceiling.
(i) After how much time screw hits the floor?
(ii) As seen from the ground, has the screw reversed its velocity before collission.

(i) $a_{SE} = \vec{a}_S - \vec{a}_E$

$= g - 2$

$a_C = -10 m/s$

$a_L = 2 m/s$



$r_{CL} + u_{CL}t + \frac{1}{2}a_{CL}t^2 = 0$

$2 + 0 + \frac{1}{2}(+12)t^2 = 0$

$2 = 6t^2$

$t = \frac{1}{\sqrt{3}}$

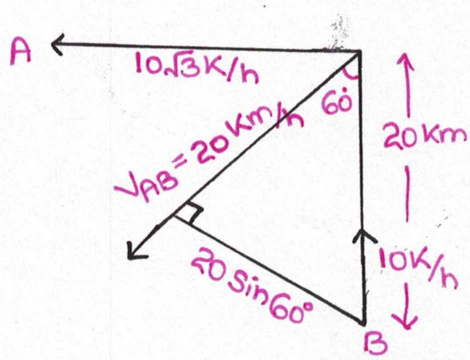
(ii) For reversal $v_c = 0$

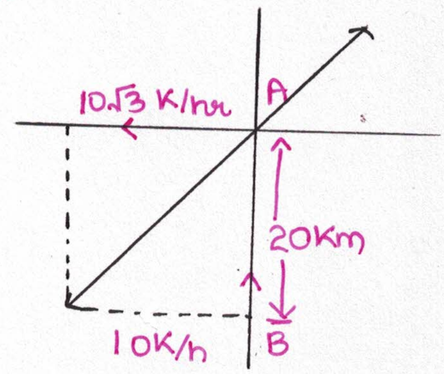
$$v_c = u_c + a_c t$$
$$0 = 5 - 10t$$
$$t = \frac{1}{2}\text{ sec.}$$

Collision takes place after reversal.

Q.) There are two ships A and B. A is moving at $10\sqrt{3}$ Km/h towards west and another is moving towards north at 10K/h. The distances b/w A and B are 20 km.
(i) Find shortest distance b/w A and B.
(ii) Find the time at which this happens.

$$v_{AB} = -10\sqrt{3}\,\hat{i} - 10\hat{j}$$





$$\text{Shortest distance} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}\text{ Km}$$

$$\text{time} = \frac{10}{20}h = \frac{1}{2}h.$$

Or

$$\vec{r}_{AB} = \vec{r}_{0AB} + \vec{u}_{AB}t$$
$$= 20\hat{j} + (-10\sqrt{3}\,\hat{i} - 10\hat{j})t$$
$$\vec{r}_{AB} = -10\sqrt{3}t\,\hat{i} + (20 - 10t)\hat{j}$$
$$Q = r_{AB}^2 = 10\sqrt{3}t^2 + (20 - 10t)^2$$

For minimum,
$$\frac{dQ}{dt} = 0$$

$$2(10\sqrt{3}t) \times 10\sqrt{3} + 2(20 - 10t)(-10) = 0$$

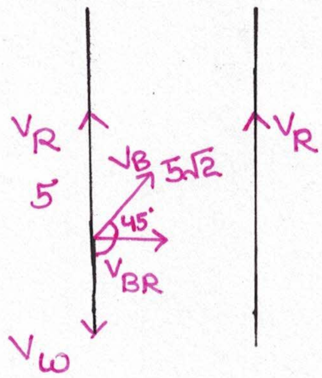
Q.) Repeat the previous problem with the foll. changed data,

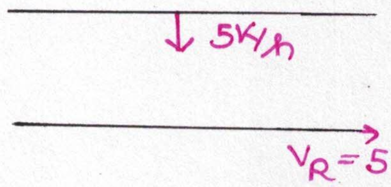
$$u_A = 10\text{ K/h}$$
$$u_B = 10\sqrt{3}\text{ K/h}$$
$$v_{AB} = -10\hat{i} - 10\sqrt{3}\hat{j}$$

$V_{AB} = -20\ K/hr.$

distance = 10 km

Q.) The velocity of a river is 5k/h north. The velocity of boat relative to river is 5K/hr east. The wind is blowing at 5Km/hr. South. In which direction will a flag mounted on the boat flutter.





$$V_B = V_{BR} + V_R$$

$$V_{WB} = V_W - V_B$$
$$= -5\hat{j} - (5\hat{i} + 5\hat{j})$$
$$= -5\hat{j} - 5\hat{i} - 5\hat{j}$$
$$(V_{WB} = -10\hat{j} - 5\hat{i})$$

$$\tan\alpha = \frac{10}{5}$$

$$\alpha = \tan^{-1} 2 \quad \text{South of west}$$

RIVER - BOAT PROBLEM

Q.) The velocity of a boat in still water is V_{BR} while a river is flowing with speed V_R. The boatman puts his efforts at an angle θ with the crossline, width of river is ω.

(i) Find the time required to cross the river (T).

(ii) Drift of the boat (L) along the river.

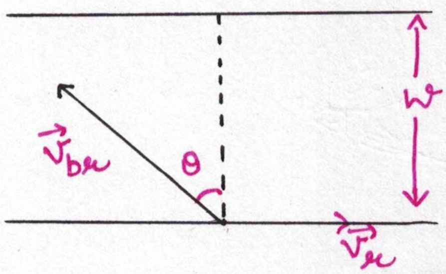
(iii) What should be the value of θ to cross in the minimum possible time.

(iv) What should be the value of θ to cross along shortest path Explain

(v) Will it always be possible to cross along shortest path?

(vi) If it is not possible to cross along shortest path then what should b θ so as to minimise the drift?

$$\vec{V_b} = \vec{V_r} + \vec{V_{br}}$$



$$\vec{V_b} = V_r\hat{i} + V_{br}\cos\theta\hat{j} - V_{br}\sin\theta\,\hat{i}$$

$$V_{bx} = V_r - V_{br}\sin\theta$$

and $S_x = V_x t$

$V_{by} = V_{br}\cos\theta$ and $S_y = V_y t$

(i) @ $t = T$, $S_y = w$

$$w = V_{br}\cos\theta . T$$

$$T = \frac{w}{V_{br}\cos\theta}$$

(ii) @ $t = T$, $S_x = L$

$$L = (V_r - V_{br}\sin\theta)T$$

$$L = (V_r - V_{br}\sin\theta)T$$

(iii) For T_{min}, $\cos\theta = 1$

$$\therefore \theta = 0^\circ$$

To cross in shortest time $\theta = 0^\circ$

and $T_{min} = \frac{W}{V_{br}}$

(iv) To cross along shortest path,

$$L = 0$$

$$(V_r - V_{br}\sin\theta) \times T = 0$$

$$\sin\theta = \frac{V_r}{V_{br}}$$

(v) No, it would not always be possible to cross along shortest path. It will be possible only when

$(V_r < V_{br})$ as $\sin\theta$ should be less than 1

$$\sin\theta < 1$$

$$\frac{V_r}{V_{br}} < 1$$

$$\boxed{V_r < V_{br}}$$

(vi) If drift is inevitable $(V_{br} < V_r)$

For L_{min}, $\frac{dL}{d\theta} = 0$

$$L = (V_r - V_{br}\sin\theta)T$$

$$L = (V_r - V_{br}\sin\theta)\frac{w}{V_{br}\cos\theta}$$

$$= \left(V_r\sec\theta - V_{br}\tan\theta \times \frac{W}{V_{br}}\right)$$

$$\frac{dL}{d\theta} = \frac{w}{V_{br}}(V_r\sec\theta . \tan\theta - V_{br}\sec^2\theta) = 0$$

multiply by $\cos^2\theta \times \frac{V_{br}}{W}$;

$$V_r \sin\theta - V_{br} = 0$$

$$\sin\theta = \frac{V_{br}}{V}$$

Q) $V_{br} = 4\,K/h$ and $V_r = 2\,K/h$. Find

(i) θ for shortest path

(ii) If time for crossing along shortest path is $2/\sqrt{3}$ hr, what is the width of river.

(iii) Minimum possible time to cross river.

(iv) How long will it take to row 2 Km upstream & back?

(i) $\sin\theta = \frac{2}{4} = \frac{1}{2}$

$$\theta = 30^\circ$$

(ii) $\frac{w}{2\cdot\cancel{4}\cancel{\sqrt{3}}/2} = \frac{\cancel{2}}{\cancel{\sqrt{3}}}$

$$w = 4\,Km$$

(iii) $T = \frac{4}{4} = 1$ hr.

(iv) For upstream $\frac{2}{2} = 1\,h = T$

$$T = \frac{2}{6} = \frac{1}{3}\text{ hr.}$$

$$T = \frac{4}{3}\text{ h.}$$

RAIN PROBLEM

Q. To a man walking at 3 K/h, the rain appears to fall vertically. If he increases his speed in the same direction to 6 K/h, it appears to fall at 45° with the vertical. What is the speed of the rain?

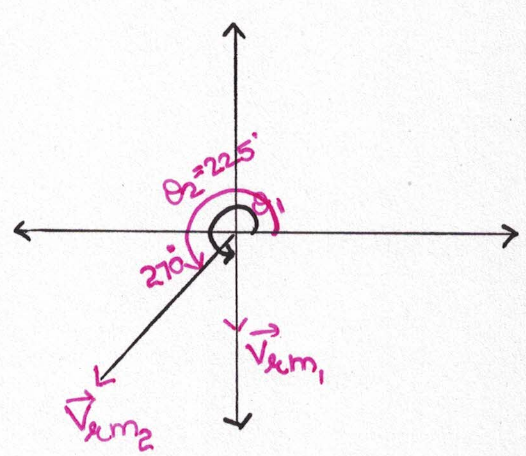
$$\vec{V}_{m_1} = 3\hat{i}$$

$$\vec{V}_r = a\hat{i} + b\hat{j}$$

$$V_{rm_1} = \vec{V}_r - \vec{V}_{m_1}$$

$$= (a-3)\hat{i} + b\hat{j}$$

$$\tan\theta_1 = \frac{b}{a-3} = \infty$$



$$a = 3$$

$\vec{V}_{rm_2} = \vec{V}_r - \vec{V}_{m_2}$

$= (a-6)\hat{i} + b\hat{j}$

$\tan\theta_2 = \dfrac{b}{a-6}$

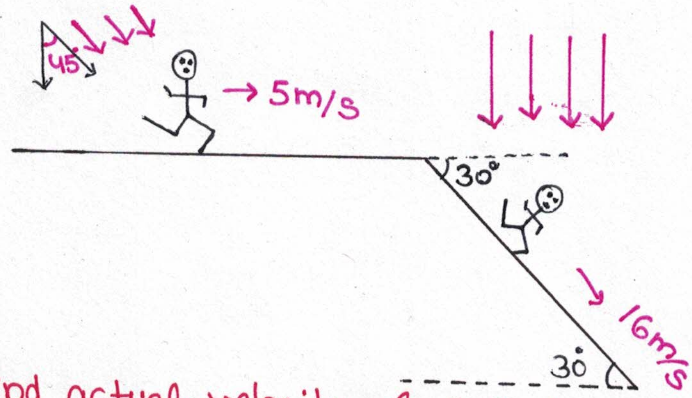
$1 = \dfrac{b}{-3}$

$b = -3$

$\vec{V}_r = 3\hat{i} - 3\hat{j}$

$|\vec{V}_r| = 3\sqrt{2}$ K/h

Q.)



Find actual velocity of rain drops.

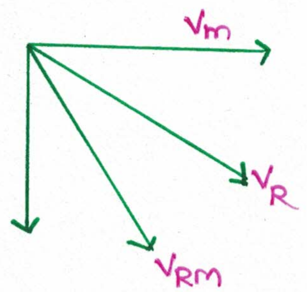
$V_{RM} = V_R - V_m$

$V_{RM} = (a-5)\hat{i} + b\hat{j}$

$V_{RM} = \tan(-45°)$

$-1 = \dfrac{b}{a-5}$

$-a + 5 = b$



$V_{RM} = V_R - V_m$

$= a + b\hat{j} - 8\sqrt{3}\hat{i} + 8\hat{j}$

$= (a - 8\sqrt{3})\hat{i} + (b+8)\hat{j}$

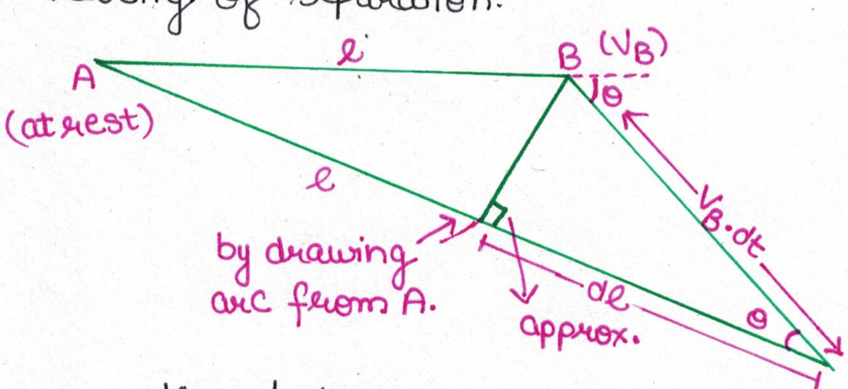
$\tan 270° = \dfrac{b+8}{a-8\sqrt{3}} = \infty$

$a = 8\sqrt{3}$

$b = 5 - 8\sqrt{3}$

VELOCITY OF SEPARATION

The rate at which the distance between two points increases is called velocity of separation.



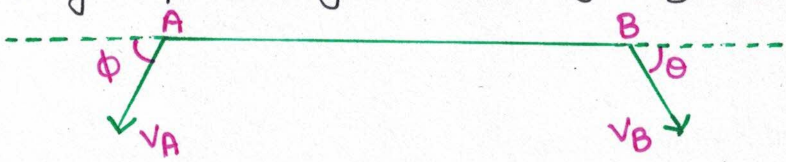
$dl = V_B \cdot d + \cos\theta$

$\dfrac{dl}{dt} = V_B \cdot \cos\theta$

From the derivation, we can see that velocity of separation can be found by taking the component of relative velocity along the line joining the two points.

If both the particles are moving then the velocity of separation can be found by considering the effects of motion of each particle one at a time.

For eg - for the given case $V_S = V_B \cos\theta + V_A \cos\phi$

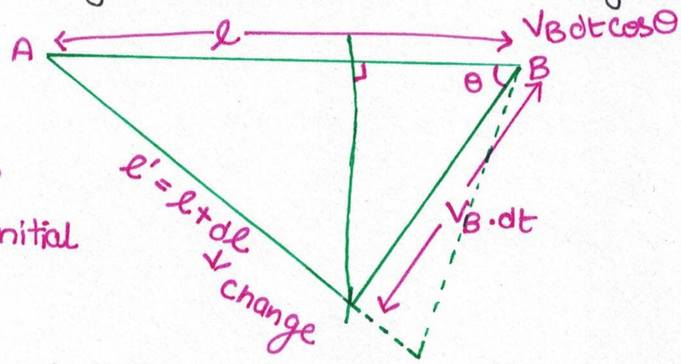


VELOCITY OF APPROACH

The rate at which the distance between any two points is decreasing is called velocity of approach. It is negative of velocity of separation.

Change = Final − Initial

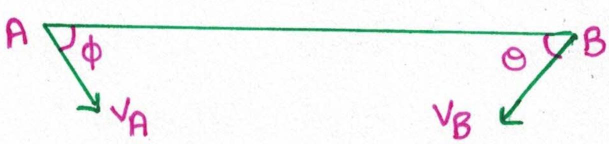
Final = change + initial



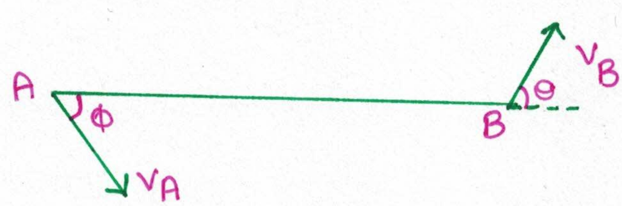
$$dl = -V_B\, dt \cos\theta$$

$$-\frac{dl}{dt} = V_B \cos\theta$$

The velocity of approach can be found by taking the component of relative velocity along the line joining the two points.



$$V_{approach} = V_B \cos\theta + V_A \cos\phi$$

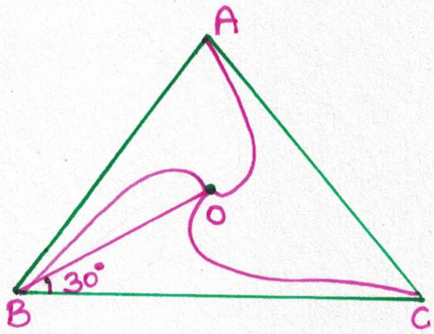


$V_{approach} = V_A \cos\phi - V_B \cos\theta$

$V_{sep.} = V_B \cos\theta - V_A \cos\phi$

Q.1 Three particles A, B & C are in an equilateral triangular configuration along a triangle of side 'a'. They begin to move with a constant speed 'u' such that A heads for B, B always heads for B & C always head for A.

(i) After how much time will they meet?
(ii) How much distance will they cover before meeting?



Velocity of approach of C towards D.

$$= U\cos 30^\circ$$

$$\omega = \frac{2}{3} a \cos 30^\circ$$

$$T = \frac{\omega}{V_{app.}}$$

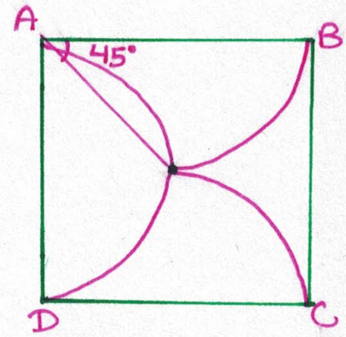
$$= \frac{2}{3} \times a \times \frac{\cancel{\sqrt{3}}}{\cancel{2}} \times \frac{1}{4} \times \frac{\cancel{2}}{\cancel{\sqrt{3}}}$$

$$= \frac{2a}{3u}$$

$\therefore$ Distance travelled = Speed $\times T$

$$= \frac{2a}{3\cancel{u}} \times \cancel{u} = \frac{2a}{3} m$$

Q) Repeat the above question for a square.



$$V_{app.AO} = u\cos 45^\circ$$

$$= \frac{4}{\sqrt{2}}$$

$$AO = \frac{a}{\sqrt{2}} = \frac{\sqrt{2}a}{2}$$

$$T = \frac{AO}{v}$$

$$= \frac{u}{\sqrt{2}} \times \frac{a\sqrt{2}}{4} = \frac{a}{u}$$

$$\text{Distance} = \frac{a\cancel{u}}{\cancel{u}} = a$$